Instructional Days: 13-14

Topic Description: This lesson introduces the linear and binary search algorithms.

**Objectives:**

**The students will be able to:**

* Describe the linear search algorithm.
* Describe the binary search algorithm.
* Explain conditions under which each search might be appropriate.

**Outline of the Lesson:**

* Tower Building Activity (55 minutes)
* Model tower building algorithm. (25 minutes)
* Model binary search (15 minutes)
* Comparison of linear and binary search (15 minutes)

**Student Activities:**

* In pairs complete the Tower Building Activity.
* Model the tower building algorithm.
* Students participate in the activity modeling binary search.

**Teaching/Learning Strategies:**

* Tower Building Activity
  + Have students complete the Tower Building Activity with their elbow partner and write their solutions in their journals.
* Model tower building activity.
  + Have students share their solutions with another elbow partner pair.
  + Have one set of students use 10 legos (or checkers or some other easily manipulated piece) to model the algorithm for solving the problem in front of the entire class.
  + Note: The solution is to start by taking half of the height of the tower and create that number of stacks of 2. Continue halving the number of stacks and doubling the height (plus one stack of any remainder) until the desired height is reached. This foreshadows binary search. (See sample solutions.)
* Model binary search.
  + Use two identical, \*sorted\*, and \*large\* stacks of index cards with words in braille and writing. Give one stack to a pair of two students and ask them to pick any word, keeping it in order.
  + Choose another pair of students to count how many times you pick a word from the stack to try and find their word.
    - Start by using a linear search. It should not take long for students to suggest that this is not a good strategy. Ask them to provide a better strategy.
    - Guide them to binary search.
  + Discuss the number of guesses required and how this is similar to the tower building problem.
* Comparison of linear and binary search.
  + Linear—start at the beginning, look at each item until you find it or there is no more data. Data can be sorted or not.
  + Binary—look at middle item, eliminate the half where the value is not located. Find the new middle element and continue the process until you find it, or there is no more data. Ask students to describe what is necessary in order to use a binary search—the list must be sorted.
  + Have students provide examples of where each type of search is appropriate and why.
    - Note that decisions often need to be made about whether to maintain lists in sorted order, provide an option for sorting should it be necessary, etc. based on the types of searches that are expected to be performed on the data.

**Resources:**

• Shasha, Dennis. The Puzzling Adventures of Doctor Ecco. Mineola, New York: Dover Publications, Inc., 1998.

• Tower Building Activity

• Sample Solutions for Tower Building Activity

**Tower Building Activity**

Donald Trump wants to build a 100 meter high tower as quickly as possible. He has unlimited resources and an unlimited budget and is willing to spend any amount to get the job done.

He has chosen to build the tower with blocks that are 100 meters long and 100 meters wide, but only 1 meter tall. The blocks interlock on top and bottom (like Lego). They cannot be stacked sideways.

Using special lifters, putting one stack on top of another stack takes one week regardless of how high the stacks are.

What is the shortest amount of time that it will take to build the tower?

Suggestions:

• Use something like Lego or a graph to help solve this problem.

• Start with a smaller tower of 5 or 10—solve a smaller problem.

• Extend that knowledge to the larger problem.

Sample Solutions for Tower Building Problem

**5 meter tower:**

|  |  |  |  |
| --- | --- | --- | --- |
| Week # | # of stacks | # of blocks in stack | remainder |
| 1 | 2 | 2 | 1 stack of 1 |
| 2 | 1 | 4 | 1 stack of 1 |
| 3 | 1 | 5 |  |

Three weeks needed for completion.

**10 meter tower:**

|  |  |  |  |
| --- | --- | --- | --- |
| Week # | # of stacks | # of blocks in stack | remainder |
| 1 | 5 | 2 | 1 stack of 1 |
| 2 | 2 | 4 |  |
| 3 | 1 | 8 | 1 stack of 2 |
| 4 | 1 | 10 |  |

Four weeks needed for completion

**100 meter tower:**

|  |  |  |  |
| --- | --- | --- | --- |
| Week # | # of stacks | # of blocks in stack | Remainder |
| 1 | 500 | 2 |  |
| 2 | 250 | 4 |  |
| 3 | 125 | 8 |  |
| 4 | 62 | 16 | 1 stack of 8 |
| 5 | 31 | 32 | 1 stack of 8 |
| 6 | 15 | 64 | 1 stack of 40 |
| 7 | 7 | 128 | 1 stack of 104 |
| 8 | 3 | 256 | 1 stack of 232 |
| 9 | 1 | 512 | 1 stack of 448 |
| 10 | 1 | 1000 |  |

Ten weeks needed for completion.

In general: The number of weeks is the smallest n such that the height of the tower is less than 2^n.